

MA3514 — Assignment No. 5

1. Determine the coefficients c_0 , c_1 , c_{-1} so that the scheme

$$u_j^{k+1} = c_{-1}u_{j-1}^k + c_0u_j^k + c_1u_{j+1}^k$$

for the solution of the equation $u_t + au_x = 0$ agrees with the Taylor series expansion of $u(x_j, t_{k+1})$ to as high an order as possible when a is a positive constant. Verify that the result is the Lax-Wendroff scheme.

2. For the following equation

$$u_{tt} = u_{xx} + 2u_x.$$

If second order central difference approximations are used to approximate u_{tt} , u_{xx} and u_x , analyze the stability property of this method.

3. Derive an explicit central difference scheme for the solution of

$$u_{xx} - (1 + 4x)^2 u_{tt} = 0, \quad 0 < x < 1, \quad t > 0,$$

with the boundary and initial conditions

$$u(x, 0) = x^2, \quad u_t(x, 0) = 0, \quad u_x(0, t) = 0, \quad u(1, t) = 1.$$

Notice that a derivative boundary condition is used at $x = 0$. You should discretize x as $x_j = (j - 0.5)\Delta x$, with $x_{n+1} = 1$, so that $\Delta x = 1/(n + 0.5)$, where n is an integer. Now, for $t_k = k\Delta t$ and $u_j^k \approx u(x_j, t_k)$,

- write down the initial conditions (for the first two time levels): u_j^0 and u_j^1 ;
- write down a formula for u_j^{k+1} (assuming that u_j^k for all j are known) and pay special attention to $j = 1$ and $j = n$.