

MA3514 — Assignment No. 4

1. For the heat equation $u_t = au_{xx}$ where a is a positive constant, show that the following method

$$\frac{1}{2\Delta t}(u_j^{k+1} - u_j^{k-1}) = \frac{a}{(\Delta x)^2}(u_{j+1}^k - 2u_j^k + u_{j-1}^k).$$

is unconditionally unstable.

2. For the heat equation $u_t = au_{xx}$ where a is a positive constant, show that the Dufort-Frankel method

$$\frac{1}{2\Delta t}(u_j^{k+1} - u_j^{k-1}) = \frac{a}{(\Delta x)^2}(u_{j+1}^k - u_j^{k+1} - u_j^{k-1} + u_{j-1}^k).$$

is unconditionally stable.

3. Consider the Crank-Nicolson method for the following initial and boundary value problem

$$\begin{aligned} u_t &= u_{xx} \quad \text{for } 0 < x < 1, \quad t > 0 \\ u_x(0, t) &= 0, \quad u(1, t) = 0, \quad t \geq 0, \\ u(x, 0) &= 1 - x, \quad 0 \leq x < 1. \end{aligned}$$

For $x_j = (j - 0.5)/4.5$ and $t_k = k/9$, write down a system of equations for the numerical solution at t_1 , namely for

$$\mathbf{v}_1 = [u_1^1, \quad u_2^1, \quad u_3^1, \quad u_4^1]^T$$

where $u_j^k \approx u(x_j, t_k)$. You do not need to solve \mathbf{v}_1 .

4. For the 2D heat equation on the unit square $\Omega = \{(x, y) \mid 0 < x < 1, 0 < y < 1\}$,

$$u_t = u_{xx} + u_{yy}, \quad (x, y) \in \Omega, \quad t > 0,$$

with the boundary condition $u(x, y, t) = x + 2y$ for $(x, y) \in \partial\Omega$ and $t \geq 0$ and the initial condition $u(x, y, 0) = 0$ for $(x, y) \in \Omega$. Write a MATLAB program for the ADI method based on the matrix notations. Choose $\Delta x = \Delta y = 1/101$ and $\Delta t = 0.01$, find the solution at $t = 0.2$, and submit the MATLAB program and a plot of the solution (by MATLAB command *image*).