

MA3514 — Assignment No. 3

1. Consider the following linear boundary value problem:

$$\begin{aligned}u'' + q(x)u &= 0, & 0 < x < L \\u'(0) &= au(0), & u'(L) = bu(L) + c,\end{aligned}$$

where L , a , b and c are given constants and q is a given function of x . Assuming u is non-zero for all $x \in [0, L]$, derive an equation for $Q = u'/u$. If Q is solved as an initial value problem from $x = 0$ to $x = L$, how do you find $u(L)$? Write down the solution u at any $x \in [0, L]$ based on Q .

2. Solve the following two-point boundary value problem by the shooting method in MATLAB.

$$\begin{aligned}u'' + [1 + x(u')^2]u &= 0, & 0 < x < 1, \\u'(0) &= 0, & u'(1) = u(1).\end{aligned}$$

You can use the secant method as the nonlinear equation solver and any Runge-Kutta method as the ODE IVP solver. Submit your MATLAB programs and a plot of the solution.

3. Consider the two-point boundary value problem

$$\begin{aligned}y'' + (1 - x)y &= x^2, & 0 < x < 1, \\y'(0) + y(0) &= 0, & y'(1) = 2.\end{aligned}$$

Use a second order finite difference method with the grid points $x_j = (j - 0.5)/3$, derive a linear system of equations for $[y_1, y_2, y_3]^T$, where $y_j \approx y(x_j)$. You do not need to solve the linear system.

4. Let $\{\phi_j\}$ be the piecewise linear functions used in the finite element method and

$$a_{kj} = \int_a^b [-\phi'_k \phi'_j + p(x)\phi_k \phi'_j + q(x)\phi_k \phi_j] dx.$$

When p and q are constants, verify the formulae:

$$\begin{aligned}a_{k,k-1} &= \frac{1}{h} - \frac{p}{2} + \frac{qh}{6} \\a_{k,k+1} &= \frac{1}{H} + \frac{p}{2} + \frac{qH}{6}\end{aligned}$$

where $h = x_k - x_{k-1}$ and $H = x_{k+1} - x_k$.

5. The eigenvalue problem

$$\begin{aligned}u'' + [\lambda - q(x)]u &= 0, & 0 < x < L, \\u(0) = u(L) &= 0,\end{aligned}$$

where q is a given function of x , can be reduced to a matrix eigenvalue problem

$$A\vec{u} = \lambda B\vec{u},$$

by the finite element method based on the piecewise linear basis functions ϕ_j introduced in the lectures. Assuming that the grid points are $x_j = jh$ for $0 \leq j \leq n+1$ and $h = L/(n+1)$, write down the matrices A and B explicitly. Approximate $q(x)$ in (x_{j-1}, x_j) by $q_{j-1/2} = q(x_{j-1} + h/2)$ if needed.