MA3514 — Assignment No. 3

1. Write down the solution of the linear boundary value problem

$$y'' + p(x)y' + q(x)y = 0 \quad \text{for} \quad a < x < b$$
$$y(a) = \alpha, \quad y(b) = \beta,$$

based on the solutions of the two initial value problems:

$$y_1'' + p(x)y_1' + q(x)y_1 = 0, \quad y_1(a) = 1, \quad y_1'(a) = 0$$
$$y_2'' + p(x)y_2' + q(x)y_2 = 0, \quad y_2(b) = 1, \quad y_2'(b) = 0.$$

2. Solve the two-point boundary value problem

$$y'' + (1 - x)y = x^2$$
$$y(0) = 1, \quad y'(1) = 2$$

with a second order finite difference method and grid size $h = 0.4$.

3. Let $\{\phi_j\}$ be the piecewise linear functions used in the finite element method and

$$a_{kj} = \int_a^b [-\phi_k'\phi_j' + p(x)\phi_k\phi_j' + q(x)\phi_k\phi_j]dx.$$

When p and q are constants, verify the formulae:

$$a_{k,k-1} = \frac{1}{h} - \frac{p}{2} + \frac{qh}{6}$$
$$a_{k,k+1} = \frac{1}{H} + \frac{p}{2} + \frac{qH}{6}$$

where $h = x_k - x_{k-1}$ and $H = x_{k+1} - x_k$.

4. Use the finite element method to solve the following boundary value problem

$$y'' + (2 - x)y = 1, \quad \text{for} \quad 0 < x < 1$$
$$y(0) = 1, \quad y(1) = 0$$

based on equally spaced grid points and the grid size $h = 1/3$.

5. The following eigenvalue problem has a sequence of eigenvalues and eigenvectors.

$$u'' + \lambda(1 + x^2)u = 0 \quad \text{for} \quad 0 < x < 1$$
$$u(0) = u(1) = 0.$$

Use shooting method (with the secant method as the nonlinear equation solver), find at least one eigenvalue and one eigenfunction. You can do this in MATLAB. Submit your own MATLAB programs and a plot of the solution.