

## MA3514 — Assignment No. 1

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1. Apply the midpoint method to the following initial value problem for  $v = v(t)$  and  $w = w(t)$ :

$$\begin{aligned}v'' &= v + wv', & t > 0 \\w' &= -w + v^2 + t, & t > 0 \\v(0) &= 1, \quad v'(0) = 0, \quad w(0) = -1,\end{aligned}$$

with the step size  $h = 0.2$ , find the approximate values of  $v(0.2)$ ,  $v'(0.2)$  and  $w(0.2)$ .

2. Consider the following system of equations,

$$\begin{aligned}x' &= -xy + 2y - \cos t \\y' &= x^2 + ty,\end{aligned}$$

for  $t > 0$ . The initial conditions are  $x(0) = -1$ ,  $y(0) = 2$ . Find the numerical solution at  $t = 0.1$ , with step size  $h = 0.1$  based on the mid-point method.

3. Apply the modified Euler's method to the following system

$$\begin{cases} x' = -y, & x(0) = 1 \\ y' = x, & y(0) = 0 \end{cases}$$

with step size  $h$ . If we denote the numerical solutions at  $t_n = hn$  by  $x_n$  and  $y_n$ , find  $\lim_{n \rightarrow \infty} (x_n^2 + y_n^2)$ .

4. For the differential equation  $y' = f(t, y)$ , the local truncation error of the general 2nd order Runge-Kutta method (with parameter  $\alpha$ ) is

$$T_{j+1} = \frac{h^3}{4} \left[ \frac{2 - 3\alpha}{3} y'''(t_j) + \alpha y''(t_j) \frac{\partial f(t_j, y(t_j))}{\partial y} \right] + O(h^4).$$

Verify this result for the special case of  $\alpha = 1$ .

5. Use the 4th order classical Runger-Kutta method to solve (by MATLAB) the Rössler system given below from  $t = 0$  to  $t = 100$  with a step size  $h = 0.01$ . The Rössler system can be written as  $d\vec{y}/dt = \vec{f}(t, \vec{y})$  where

$$\vec{y} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \vec{f} = \begin{pmatrix} -y - z \\ x + 0.2y \\ 0.2 + xz - 5.7z \end{pmatrix}.$$

Use the following initial conditions:

$$x(0) = -2 \quad y(0) = 7.5, \quad z(0) = 1.2.$$

Use MATLAB to plot six figures: (1).  $t$  vs.  $x$  ( $t$  is the horizontal axis); (2)  $t$  vs.  $y$ ; (3)  $t$  vs.  $z$ ; (4)  $x$  vs.  $y$ ; (5)  $x$  vs.  $z$ ; (6)  $y$  vs.  $z$ . Hand in the plots and your MATLAB program.

6. The following is another third order Runge-Kutta method for  $y' = f(t, y)$ :

$$\begin{aligned}k_1 &= f(t_j, y_j) \\k_2 &= f\left(t_j + \frac{h}{2}, y_j + \frac{h}{2}k_1\right) \\k_3 &= f\left(t_j + \frac{3}{4}h, y_j + \frac{3}{4}hk_2\right) \\y_{j+1} &= y_j + \frac{h}{9}(2k_1 + 3k_2 + 4k_3).\end{aligned}$$

Describe an embedded Runge-Kutta method based on the above third order method and a related second order Runge-Kutta method. Use this method to calculate the first time step  $t_1$  and the numerical solution  $y_1$  for

$$\begin{aligned}y' &= y - ty^2, \quad t > 0 \\y(0) &= 1,\end{aligned}$$

based on the initial step size  $h = 0.1$  and the error tolerance  $\epsilon = 10^{-3}$ .