City University of Hong Kong

Course code & title:	MA3513 Elementary Numerical Methods
Session:	Semester A, 2004-2005
Date:	December 16, 2004
Time:	18:30 pm — 21:30 pm
Time allowed:	Three hours

This paper has THREE pages. (Including this page)

Instructions to candidates:

- Attempt all **SEVEN** questions.
- Start each question on a new page.
- Show all working.

Materials, aids & instruments permitted to be used during examination:

- Non-programmable portable battery operated calculator.

1. Consider the function
$$f(x) = x^2 + \frac{1}{x} - 3x.$$
   (a) (5 marks) Is there a solution of $f(x) = 0$ in the interval $(0.5, 1)$? Why?
   (b) (5 marks) If Newton's method is used to solve $f(x) = 0$ with the initial guess $x_0 = 1$, find x_1.
   (c) (5 marks) If the secant method is used to solve $f(x) = 0$ with the initial guesses $x_0 = 1$ and $x_1 = 0.5$, find x_2.

2. (15 marks) Find the LU decomposition with *partial pivoting* for
$$A = \begin{bmatrix} -3/2 & -1 & 0 \\ 3 & 2 & 6 \\ -1 & 4/3 & -6 \end{bmatrix}.$$

3. (15 marks) The following numerical integration formula
$$\int_0^1 \frac{1}{\sqrt{x}} f(x)\, dx \approx c_0 f(0) + c_1 f(x_1)$$
always uses the value of f at $x = 0$ and it is exact when f is any polynomial of degree less than or equal to 2. Find c_0, c_1 and x_1.

4. (10 marks) Let $P_{10}(x)$ be the polynomial (degree ≤ 10) interpolating the eleven points (x_j, y_j) given in the following table:

x_j	0	1	2	...	9	10
y_j	0^2	1^2	2^2	...	9^2	101

   Find $P_{10}(11)$.

5. (15 marks) Let A be a 5×3 matrix with full column rank (i.e. $rank(A) = 3$) and it has a QR factorization: $A = QR$, where
$$Q = I - \frac{2}{v^T v} v v^T, \quad v = [1, 0, -1, 0, 1]^T.$$
   For the column vector $b = [1, 2, 3, 2, 1]^T$, find
$$\min_{x \in R^3} ||Ax - b||.$$

6. (15 marks) Let us assume that we know α_j, β_j for $j = 0, 1, 2, 3$. They are related to c_0, c_1, ... c_7 as follows:

$$\alpha_j = c_0 + c_2\omega_4^j + c_4\omega_4^{2j} + c_6\omega_4^{3j}, \quad \beta_j = c_1 + c_3\omega_4^j + c_5\omega_4^{2j} + c_7\omega_4^{3j},$$

where $\omega_4 = e^{i2\pi/4} = i$. Next, we define $\hat{f}_k$ by

$$\begin{aligned} \hat{f}_0 &= 2c_0, \qquad \hat{f}_1 = c_0 + c_1 \\ \hat{f}_2 &= 2c_2, \qquad \hat{f}_3 = c_2 + c_3, \\ \hat{f}_4 &= 2c_4, \qquad \hat{f}_5 = c_4 + c_5, \\ \hat{f}_6 &= 2c_6, \qquad \hat{f}_7 = c_6 + c_7. \end{aligned}$$

Find the formulas for

$$f_j = \sum_{k=0}^{7} \hat{f}_k e^{i2\pi jk/8} \quad \text{for} \quad j = 0, 1, ..., 7,$$

in terms of α_j and β_j.

7. Let A be a 3×3 matrix and we assume that A has a LU decomposition. Thus, $A = LU$ for a unit lower triangular matrix L and an upper triangular matrix U. Let $\hat{A}$ be the matrix given as $\hat{A} = UL$.

   (a) (8 marks) Show that the eigenvalues of A and $\hat{A}$ are the same.

   (b) (7 marks) If the $(3, 1)$ entry of A is zero, show that the $(3, 1)$ entry of $\hat{A}$ is also zero.