

1. For a real symmetric positive definite matrix A given as

$$A = \begin{bmatrix} a & b^T \\ b & C \end{bmatrix}$$

where a is the $(1, 1)$ entry of A , b is an $(n - 1) \times 1$ column vector, b^T is the transpose of b and C is an $(n - 1) \times (n - 1)$ matrix, the first step of Cholesky decomposition gives

$$A = \begin{bmatrix} \sqrt{a} & 0 \\ b/\sqrt{a} & I \end{bmatrix} \begin{bmatrix} 1 & \\ & Z \end{bmatrix} \begin{bmatrix} \sqrt{a} & b^T/\sqrt{a} \\ 0 & I \end{bmatrix},$$

where $Z = C - bb^T/a$. Show that Z is also symmetric positive definite.

2. Find the values of β , such that the matrix A below is positive definite.

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 13 & 23 \\ 4 & 23 & \beta \end{bmatrix}.$$

3. Find the LU decomposition with partial pivoting (i.e. PA=LU) for the matrices below.

$$(a). \quad A = \begin{bmatrix} 1 & 1 & 7 \\ 0 & 1 & 3 \\ -2 & 2 & 4 \end{bmatrix}, \quad (b). \quad A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{bmatrix}.$$

4. For a vector $x = (x_1, x_2, \dots, x_m)^T$, we define its p -norm as

$$\|x\|_p = (|x_1|^p + |x_2|^p + \dots + |x_m|^p)^{1/p}.$$

What is $\|x\|_\infty$? The matrix p -norm is defined as

$$\|A\|_p = \max_{x \neq 0} \frac{\|Ax\|_p}{\|x\|_p}.$$

For the matrix

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix},$$

calculate $\|A\|_1$, $\|A\|_2$, $\|A\|_\infty$.

5. Let A be a tridiagonal matrix

$$A = \begin{bmatrix} a_1 & c_1 & & & \\ b_1 & a_2 & c_2 & & \\ & b_2 & \ddots & \ddots & \\ & & \ddots & \ddots & c_{n-1} \\ & & & b_{n-1} & a_n \end{bmatrix}.$$

- (a) Write a MATLAB function for its LU decomposition (without partial pivoting) using only three vectors. For input, you have the three vectors a , b and c that define the matrix A . For output, the entries of L and U are stored in b and a , c , respectively. The original values of a , b and c are destroyed. That is

$$L = \begin{bmatrix} 1 & & & & & & \\ b_1 & 1 & & & & & \\ & b_2 & \ddots & & & & \\ & & \ddots & \ddots & & & \\ & & & b_{n-1} & 1 & & \\ & & & & & & \end{bmatrix}, \quad U = \begin{bmatrix} a_1 & c_1 & & & & & \\ & a_2 & c_2 & & & & \\ & & \ddots & \ddots & & & \\ & & & \ddots & \ddots & & \\ & & & & \ddots & c_{n-1} & \\ & & & & & & a_n \end{bmatrix}.$$

- (b) Write a MATLAB function for solving $Ly = f$ using only two vectors. One vector for the matrix L and the other vector f . The solution y is written in f .
- (c) Write a MATLAB function for solving $Ux = y$ using only three vectors. Two vectors for the matrix U and the vector y . The solution x is written in y .
- (d) Write a main program in MATLAB that solves $Ax = f$ for a tridiagonal matrix based on its LU decomposition. Test you program on the following 7×7 system:

$$\begin{bmatrix} 1 & 6 & & & & & \\ 6 & 2 & 5 & & & & \\ & 5 & 3 & \ddots & & & \\ & & \ddots & \ddots & 1 & & \\ & & & & 1 & 7 & \end{bmatrix} x = \begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$