

## MA3513 — Assignment # 5

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1. For a given sequence  $c_0, c_1, \dots, c_n$ , the discrete cosine transform produces the new sequence  $f_0, f_1, \dots, f_n$  by

$$f_j = \sum_{k=0}^n c_k \cos\left(\frac{jk\pi}{n}\right) \quad \text{for } j = 0, 1, \dots, n.$$

Write down a matrix  $T$  such that

$$\begin{bmatrix} f_0 \\ f_1 \\ \vdots \\ f_n \end{bmatrix} = T \begin{bmatrix} c_0 \\ c_1 \\ \vdots \\ c_n \end{bmatrix}$$

Use MATLAB, find the relationship between

$$T \quad \text{and} \quad \frac{n}{2}T^{-1}.$$

You do not need to prove the result.

2. Let  $f_j$  for  $0 \leq j < N$ , be the discrete Fourier transform (DFT) of  $\hat{f}_k$  for  $0 \leq k < N$ , given by

$$f_j = \sum_{k=0}^{N-1} \hat{f}_k e^{i2\pi jk/N}. \quad (1)$$

If  $N$  is a multiple of 3,  $\alpha_j, \beta_j$  and  $\gamma_j$  for  $0 \leq j < N/3$  are the DFTs of  $\hat{f}_{3k}, \hat{f}_{3k+1}$  and  $\hat{f}_{3k+2}$  for  $0 \leq k < N/3$ , express  $f_j, f_{j+N/3}$  and  $f_{j+2N/3}$  in terms of  $\alpha_j, \beta_j$  and  $\gamma_j$ .

3. Consider the DFT given in (1). If the coefficients  $\hat{f}_k$  for  $0 \leq k < N$ , are real, we can calculate  $f_j$  by a different DFT of length  $N/2$ . We define

$$g_j = \sum_{k=0}^{N/2-1} [\hat{f}_{2k} + i\hat{f}_{2k+1}] e^{i4\pi jk/N}, \quad h_j = \sum_{k=0}^{N/2-1} [\hat{f}_{2k} - i\hat{f}_{2k+1}] e^{i4\pi jk/N} \quad \text{for } j = 0, 1, \dots, \frac{N}{2} - 1.$$

Show that  $h_j = \overline{g_{N/2-j}}$  (the overline denotes the complex conjugate) for  $0 \leq j < N/2$ . Find the formula for  $f_j$  in terms of  $g_j$ .

4. The function

$$f(x) = \frac{2}{5 - 3 \sin(x)}$$

is  $2\pi$  periodic. We can discretize  $x$  and  $f$  by

$$f_j = f(x_j), \quad x_j = \frac{j2\pi}{N}, \quad j = 0, 1, 2, \dots, N - 1.$$

Write a MATLAB program (based on `fft` and `ifft`) to calculate its approximate second order derivative

$$f''(x_j), \quad j = 0, 1, 2, \dots, N - 1.$$

Also, find the  $f''$  analytically and calculate the maximum error (in MATLAB)

$$E_N = \max_{0 \leq j < N} |f''(x_j)^{(analytic)} - f''(x_j)^{(numerical)}|$$

for  $N = 4, 8, 16, 32$ . Submit the MATLAB program and the results of  $E_N$ .