

1. Find approximate values of

$$I = \int_1^2 \frac{e^{-x^2}}{1+x} dx$$

based on

- the composite trapezoidal rule with grid size $h = 1/6$;
 - the composite Simpson's method with a grid size of $h = 1/6$;
 - the three point Gauss-Legendre formula.
2. Similar to Simpson's rule based on quadratic polynomial interpolation, we can use polynomial interpolations of higher degree for numerical integration. For $x_j = x_0 + jh$, derive the following formula (called Simpson's $\frac{3}{8}$ rule)

$$\int_{x_0}^{x_3} f(x) dx \approx \frac{3h}{8} [f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)]$$

based on $f(x) \approx P_3(x)$, where P_3 interpolates f at x_0, x_1, x_2 and x_3 .

3. Design an integration formula of the form:

$$\int_0^1 x^{-1/3} f(x) dx \approx c_0 f(x_0) + c_1 f(x_1)$$

such that the above formula has no error when f is a polynomial with a degree as high as possible.

4. Write a MATLAB program to calculate the zeros of the Legendre polynomial $L_n(x)$ for a given integer $n \geq 2$ by Newton's method. You need the recurrence formula to evaluate $L_n(x)$ and the following formula for $L'_n(x)$

$$L'_n(x) = \frac{n}{x^2 - 1} [xL_n(x) - L_{n-1}(x)].$$

For the j -th zero of $L_n(x)$, you can use the following initial guess:

$$x_j^{(0)} = \cos \frac{(j - 0.25)\pi}{n + 0.5}.$$

Submit your program and the results for $n = 8$ and $n = 9$.

5. Describe an adaptive trapezoidal method based on the same idea as adaptive Simpson's method. Write a MATLAB code for your adaptive trapezoidal method and test your code for $\int_0^2 \sin(x^4)/(1+x) dx$ with error tolerance $\epsilon = 0.0001$.