

## MA3513 ELEMENTARY NUMERICAL METHODS, ASSIGNMENT NO. 2

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1. Write some simple programs in MATLAB and find the solution of

$$x^3 - \cos(x) = 0$$

in the interval  $(0, \pi/2)$  using (a) bisection method and (b) Newton's method. Submit your programs and the results.

2. Write a MATLAB program for the secant method. Besides the two initial guesses  $x_0$  and  $x_1$ , the program should also input  $\epsilon$  and  $\delta$ , such that it returns  $x_{k+1}$  when

$$\left| \frac{x_{k+1} - x_k}{x_{k+1}} \right| < \epsilon, \quad \text{or} \quad |f(x_{k+1})| < \delta.$$

Use the program to find a zero of the function

$$f(x) = (x - 1)(x - 2)\dots(x - 20) - 10^{-8}x^{19}$$

in the interval  $[20, 21]$ .

3. For Newton's method, find  $C_1$  and  $C_2$  by a Taylor expansion around  $x_*$ , such that

$$x_{k+1} - x_* = C_1(x_k - x_*)^2 + C_2(x_k - x_*)^3 + \dots$$

where  $x_*$  is the exact solution of  $f(x) = 0$  and  $C_1, C_2$  are functions of  $x_*$  only.

4. If  $x_*$  is a double zero of  $f$ , we have

$$0 = f(x_*) = f'(x_*) \neq f''(x_*).$$

We can also write  $f(x) = (x - x_*)^2 g(x)$  for some function  $g$ . For Newton's method given as

$$x_{k+1} = \phi(x_k) = x_k - \frac{f(x_k)}{f'(x_k)},$$

show that  $\phi'(x_*) \neq 0$ . We also define the modified Newton's method as

$$x_{k+1} = \varphi(x_k) = x_k - 2 \frac{f(x_k)}{f'(x_k)}.$$

Show that  $\varphi'(x_*) = 0$ .

5. Let  $e_n > 0$  for  $n = 0, 1, 2, \dots$  and  $e_n \rightarrow 0$  as  $n \rightarrow \infty$ . If for some  $C > 0$ ,

$$e_{n+1} = C e_n^2 e_{n-1} \quad \text{for } n = 1, 2, 3, \dots$$

find  $\alpha > 1$  and  $\lambda > 0$ , such that

$$e_{n+1} = \lambda e_n^\alpha.$$